Multivariate B-splines and applications

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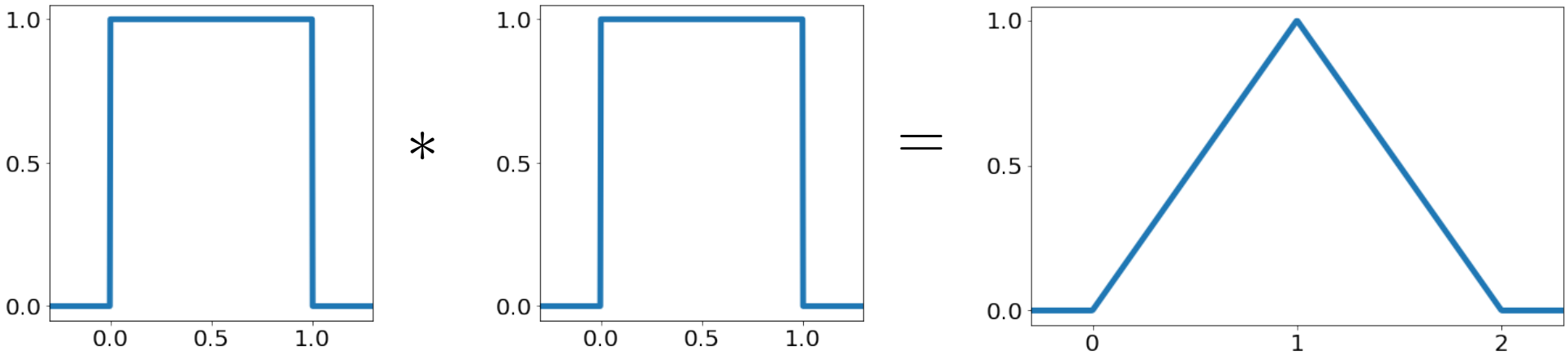
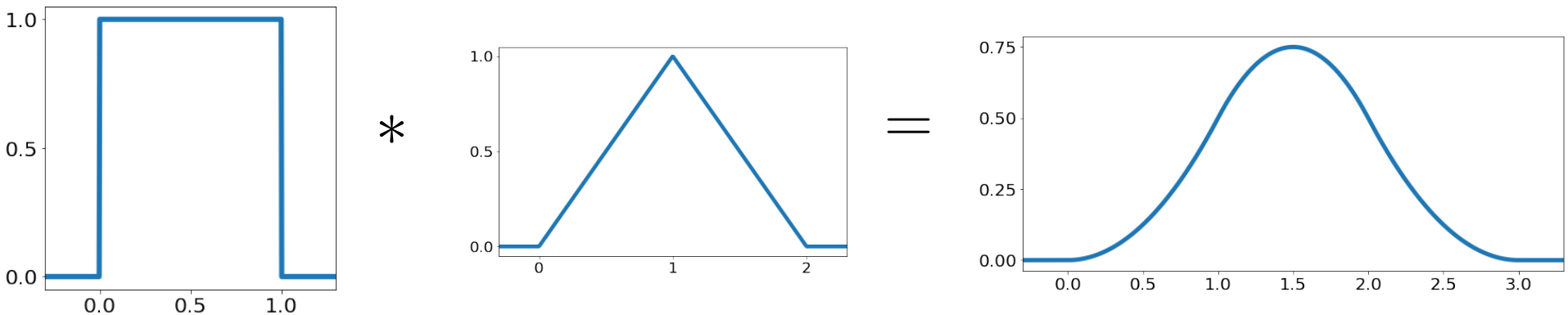
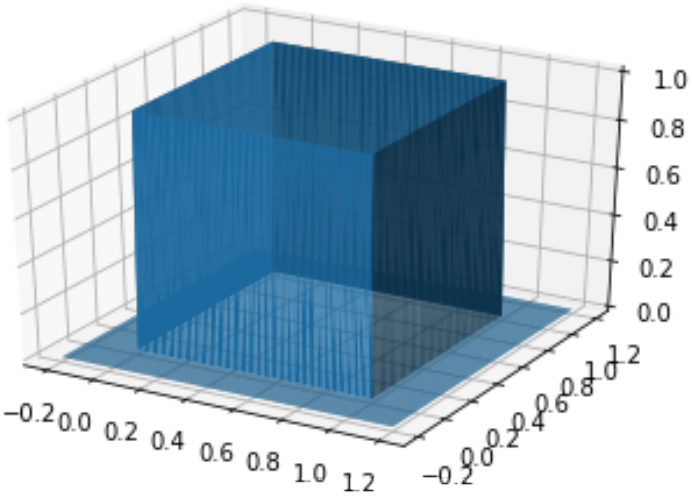
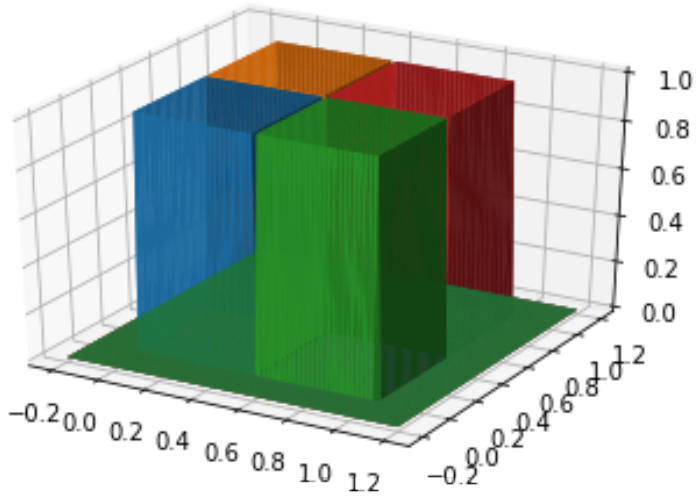
Univariate B-splines

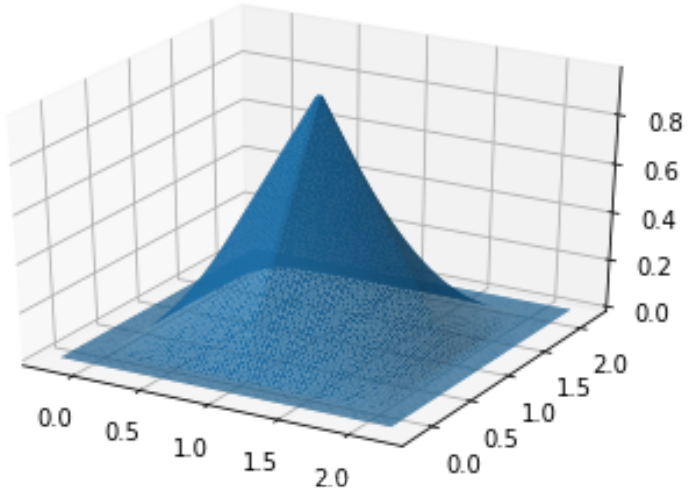




The classical multivariate B-splines



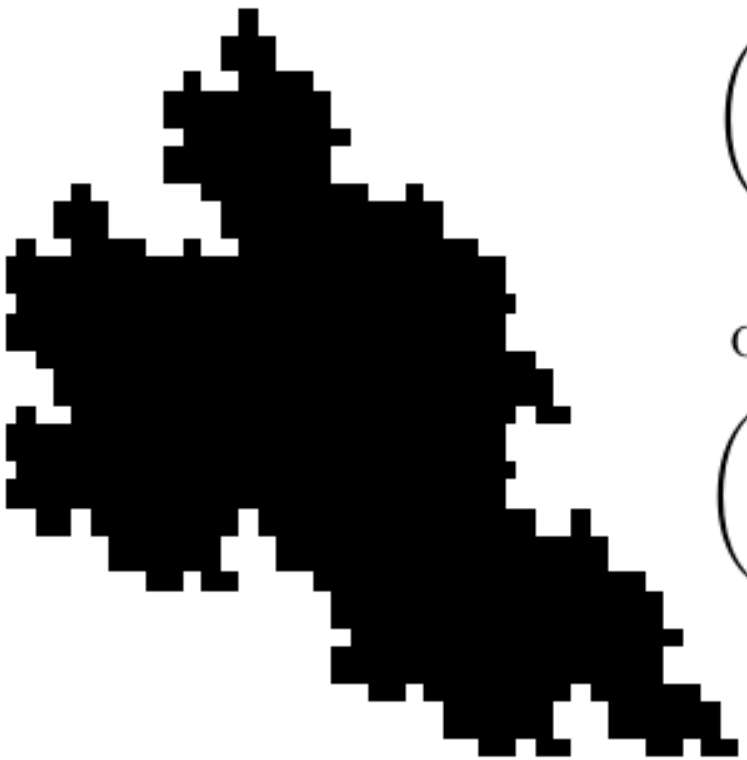




Tiles

G is a tile if

- (self-similarity) $G = \bigcup_{d_i-\text{digits}} M^{-1}(G + d_i)$
- (tiling property) integer shifts of G cover the entire space in one layer

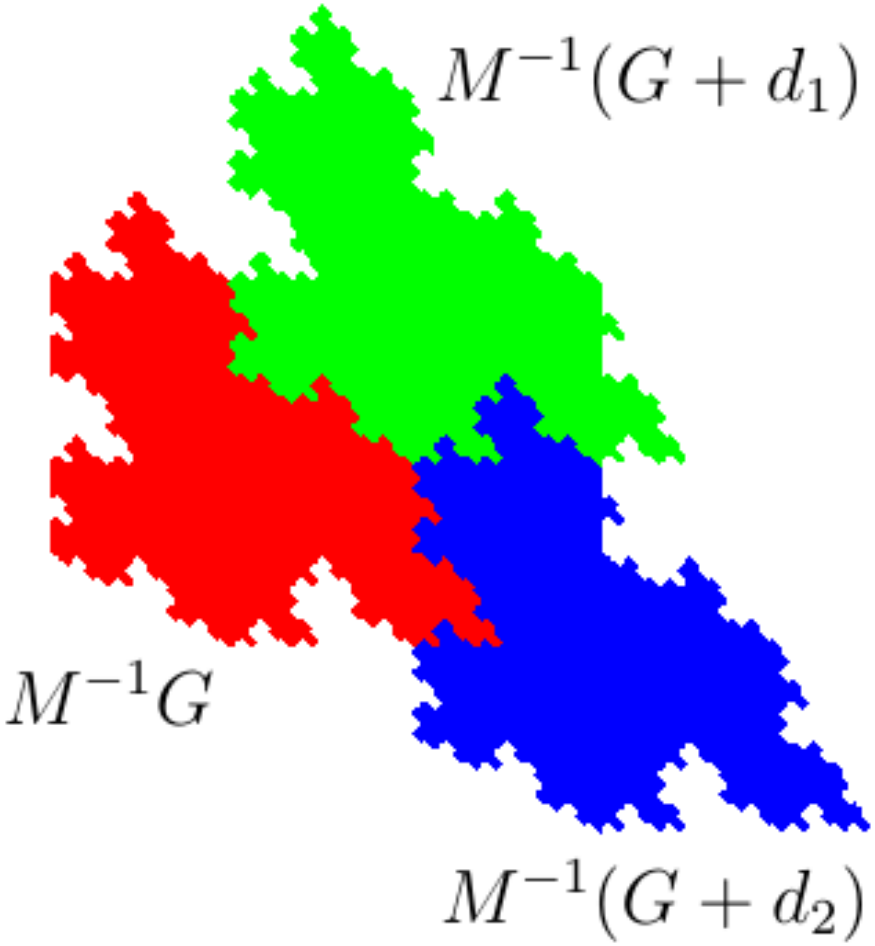


$$\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$$

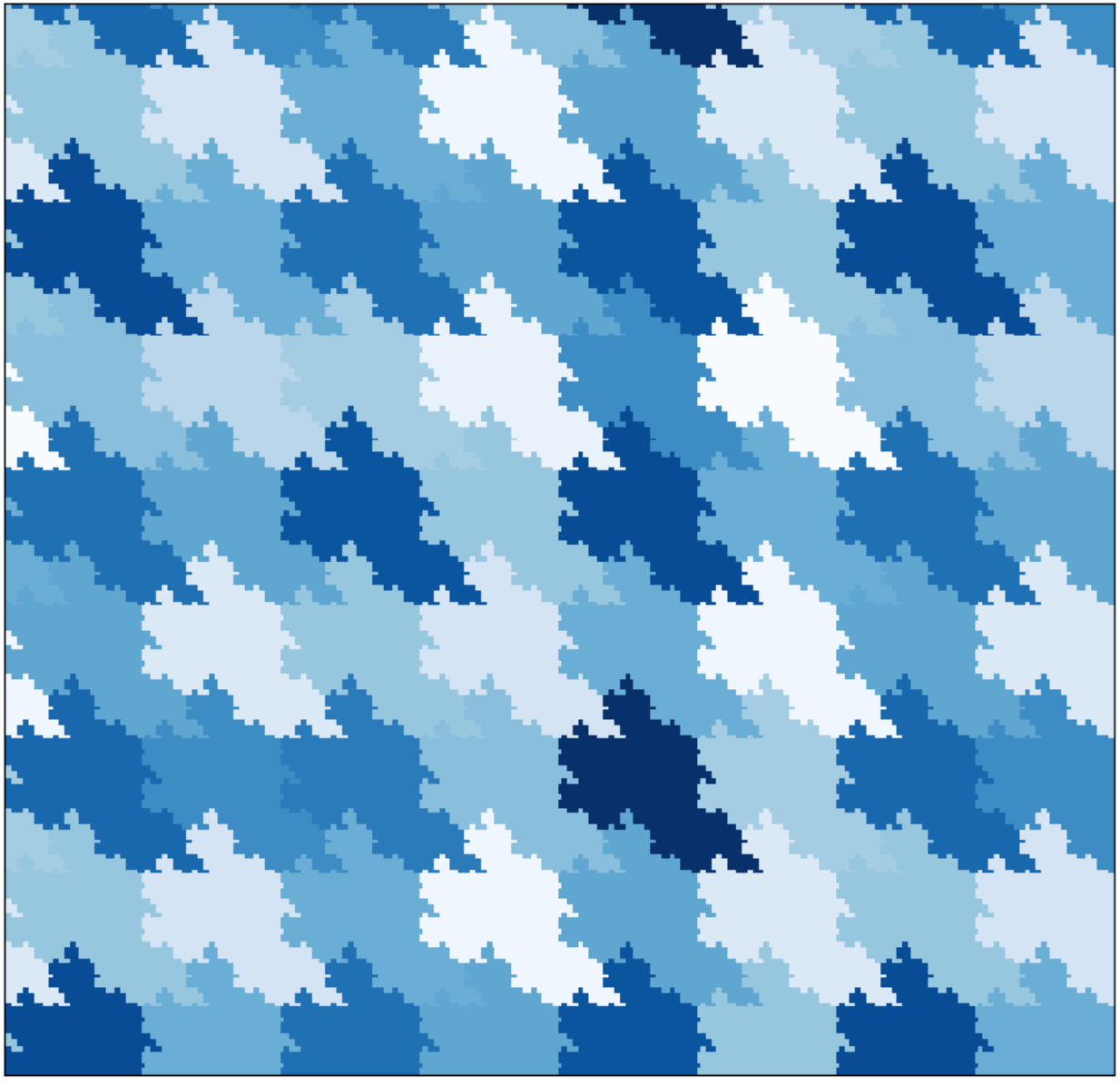
$$\text{digits}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Self-similarity: the partition into 3 parts



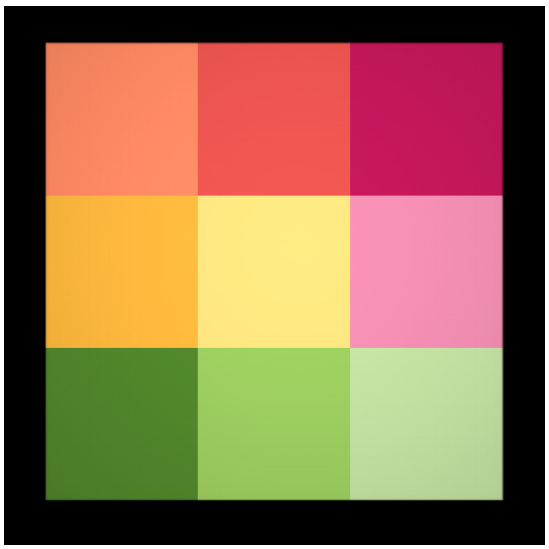
Tiling



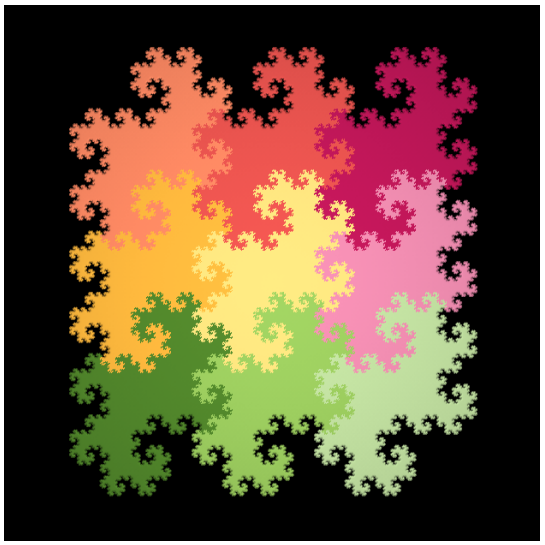
Classification of tiles with 2 digits on the plane

Theorem

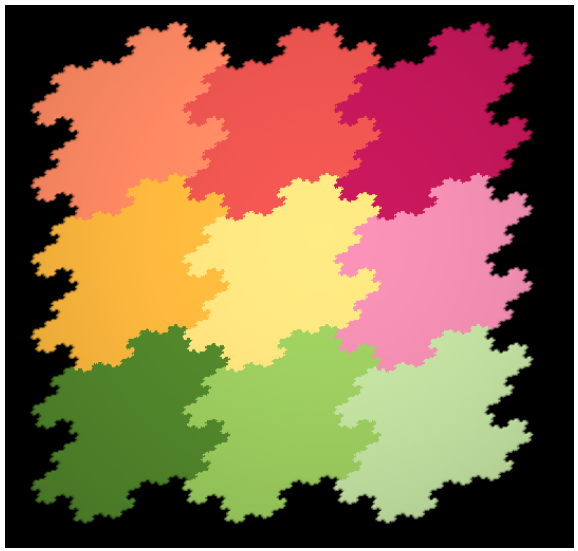
Up to an affine similarity, there are exactly three different tiles with 2 digits on the plane.



Square

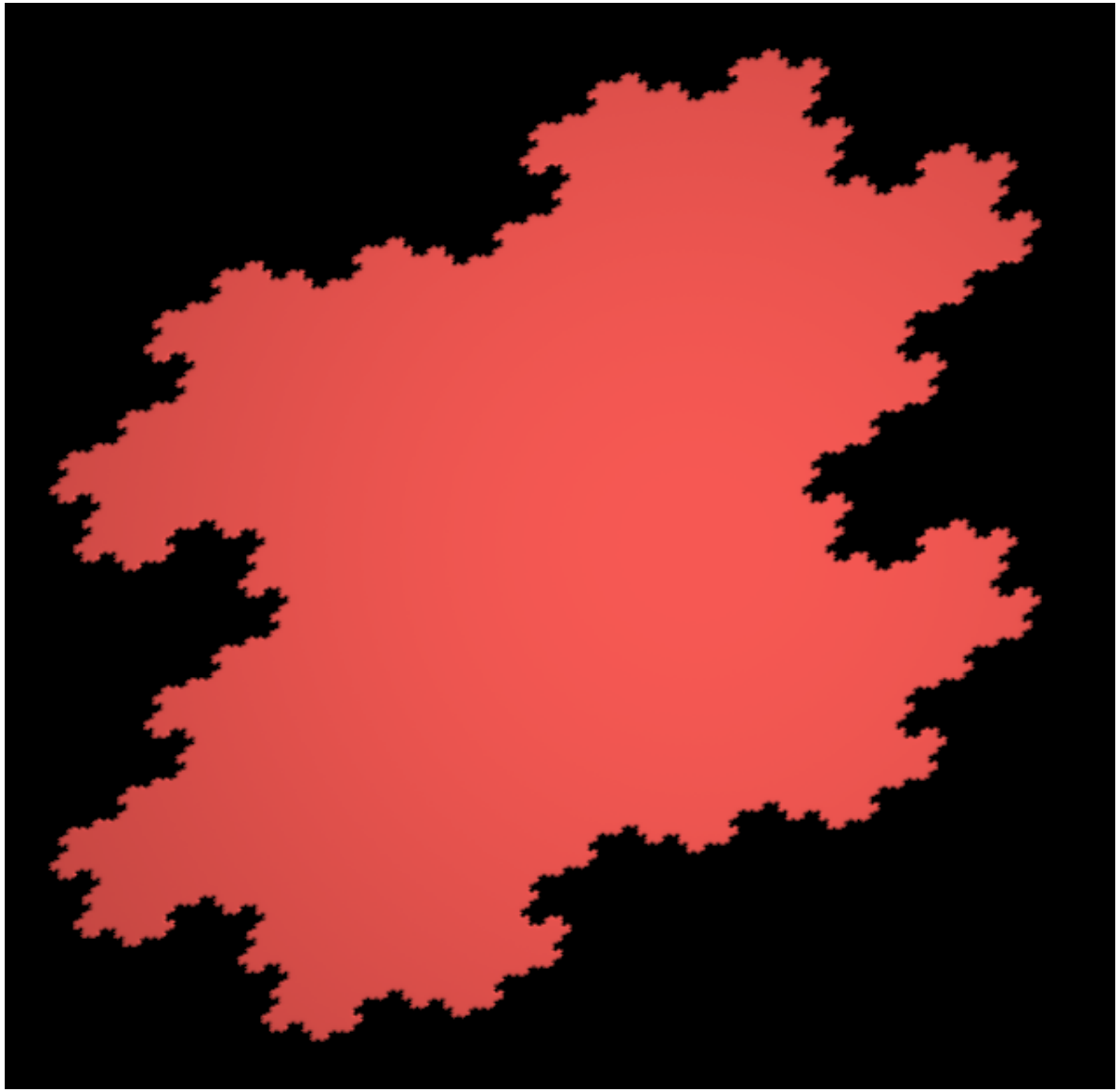


Dragon

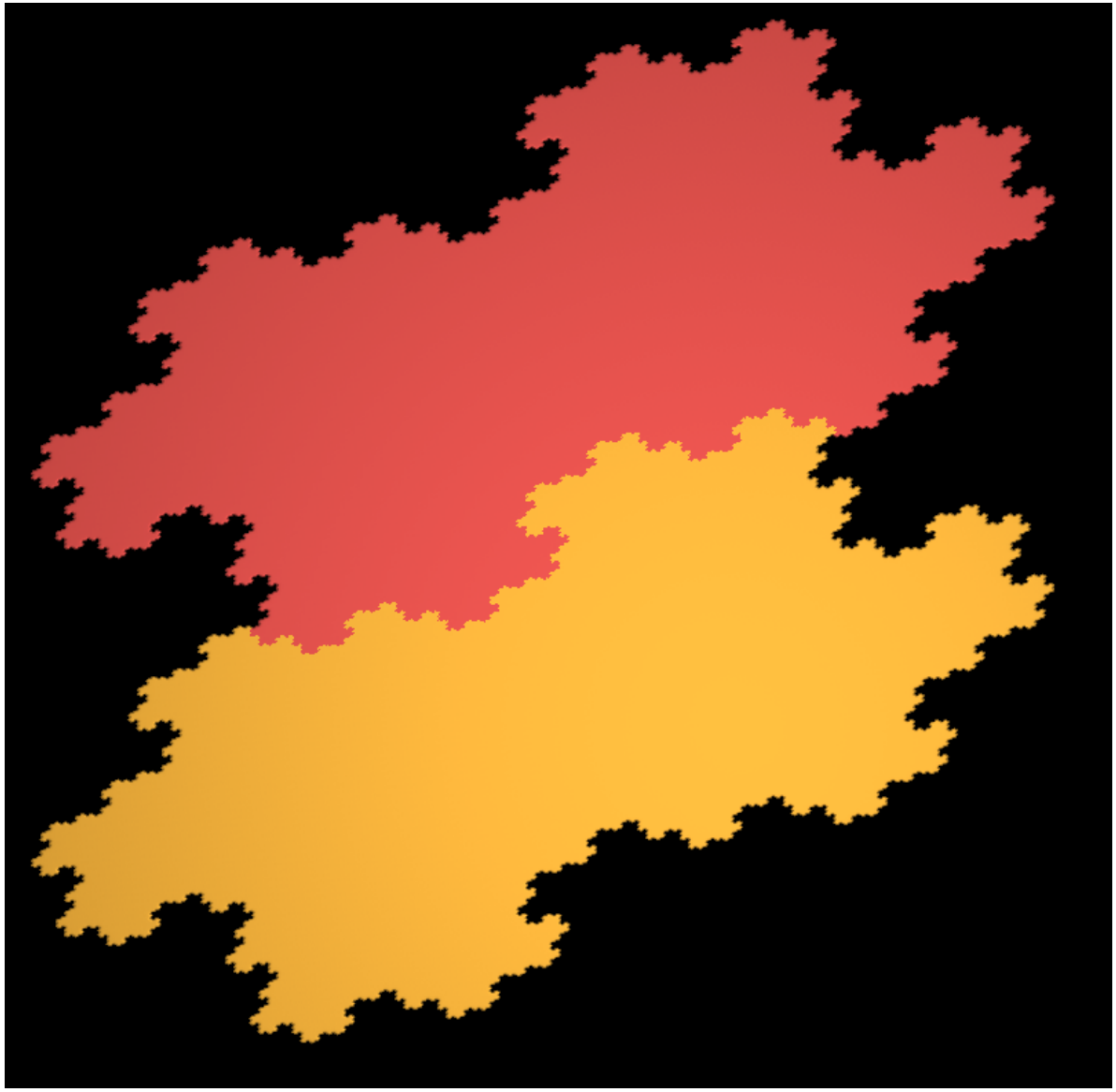


Bear

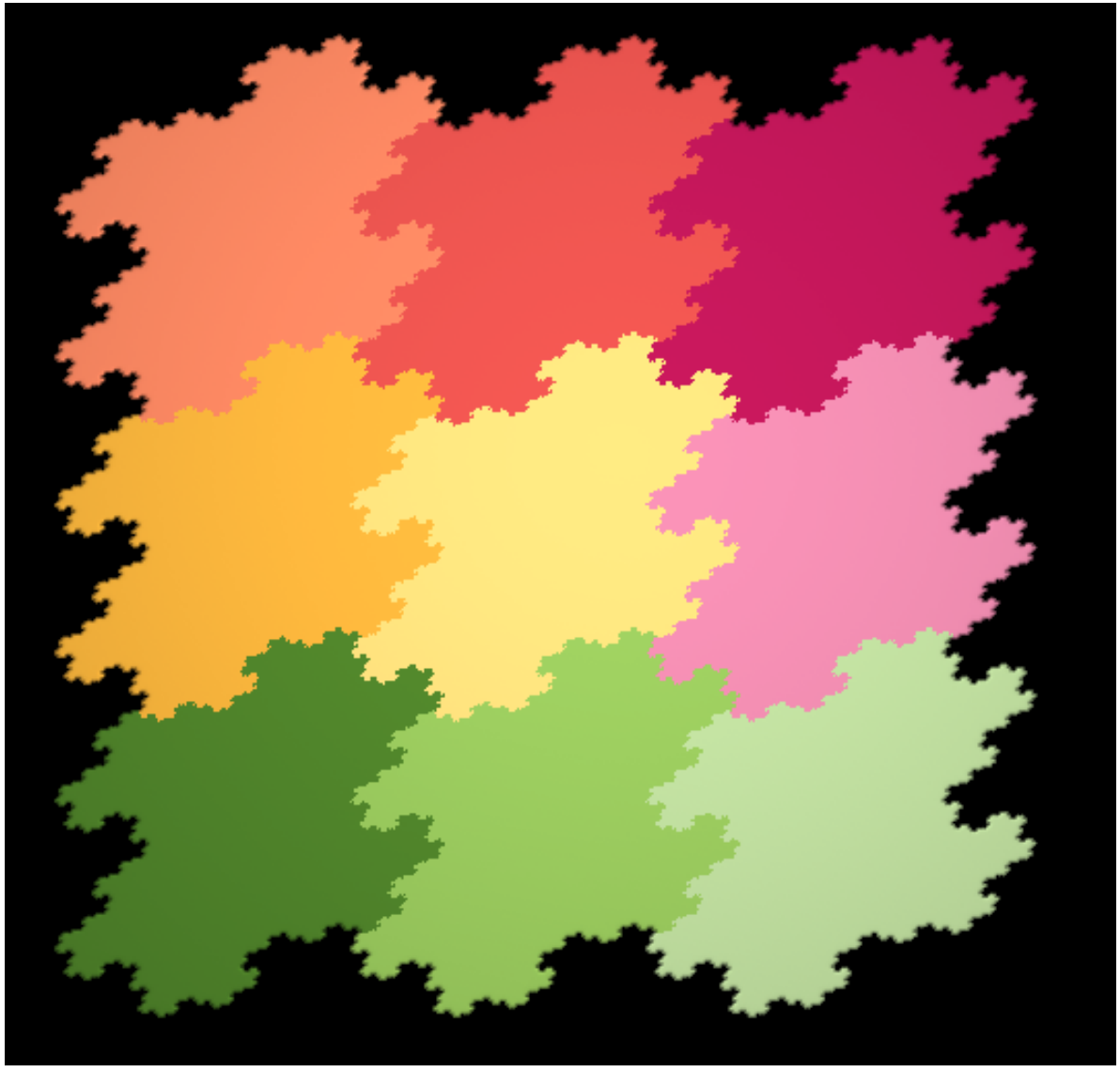
This is a Bear tile



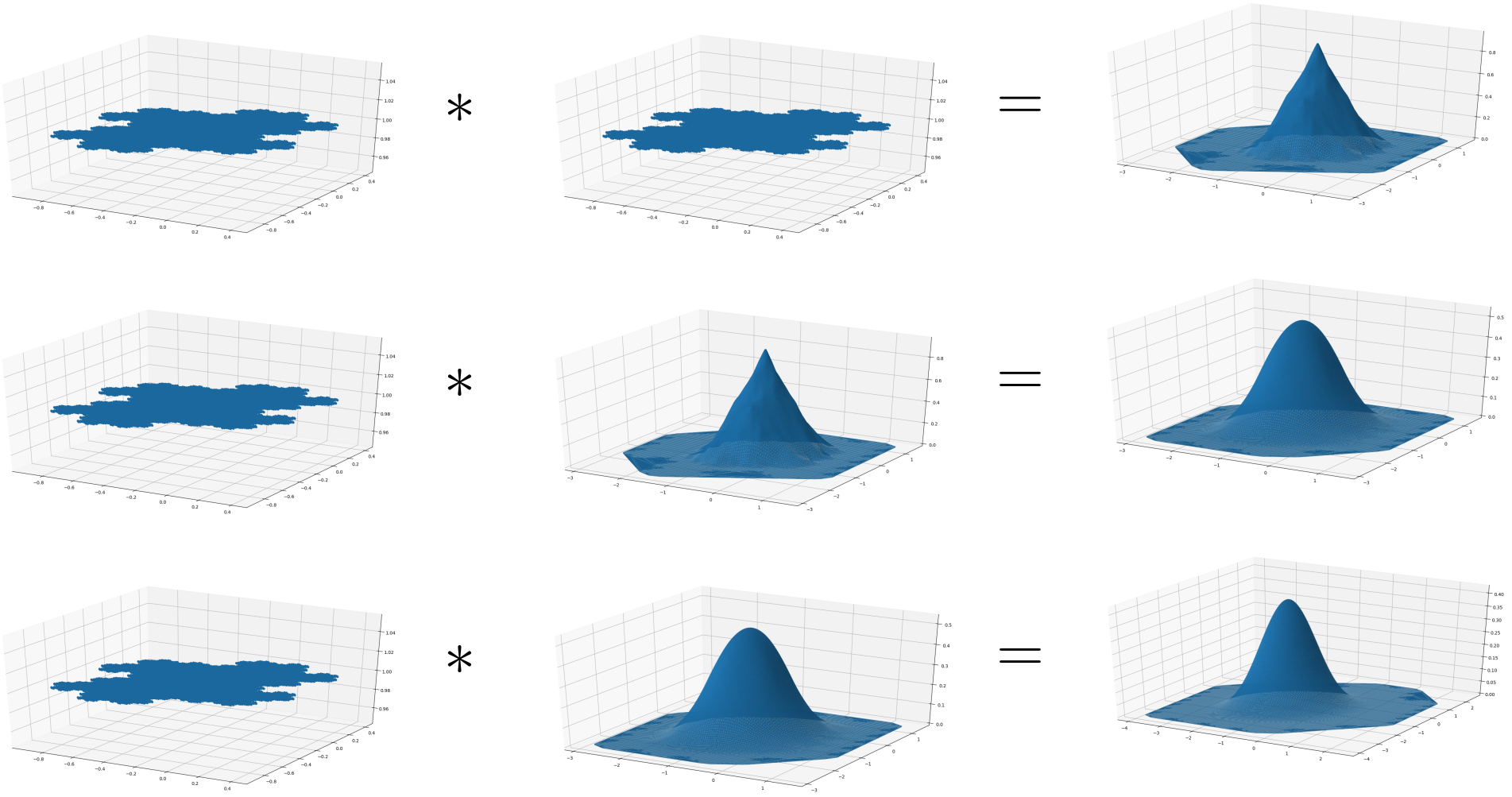
It is self-similar



Its integer shifts tile the plane

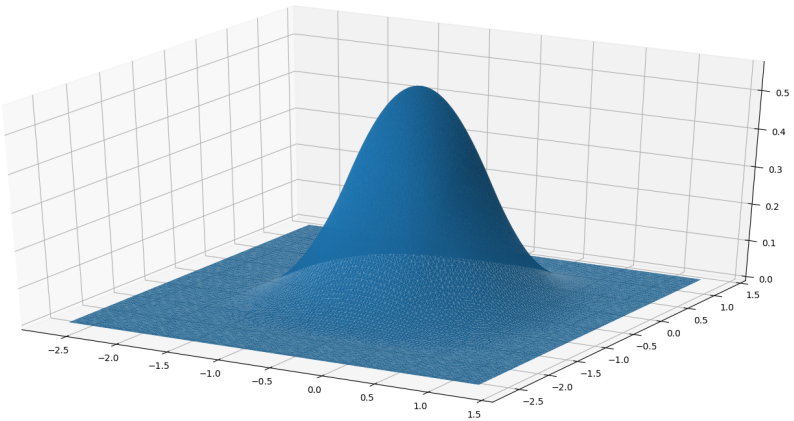


And we use it to construct Bear-splines

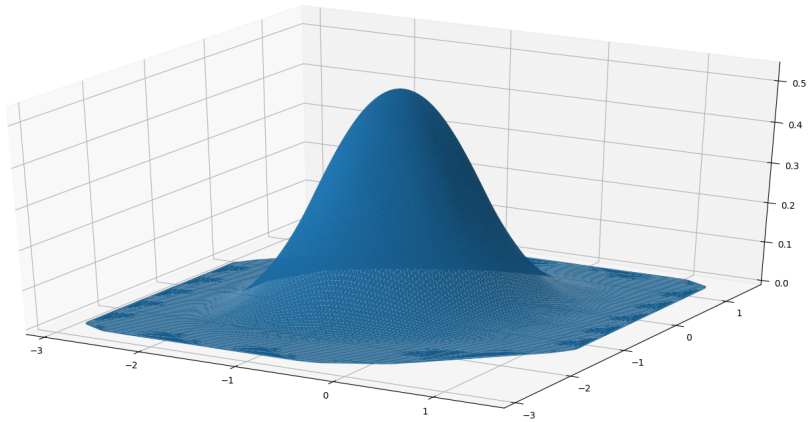


Main result

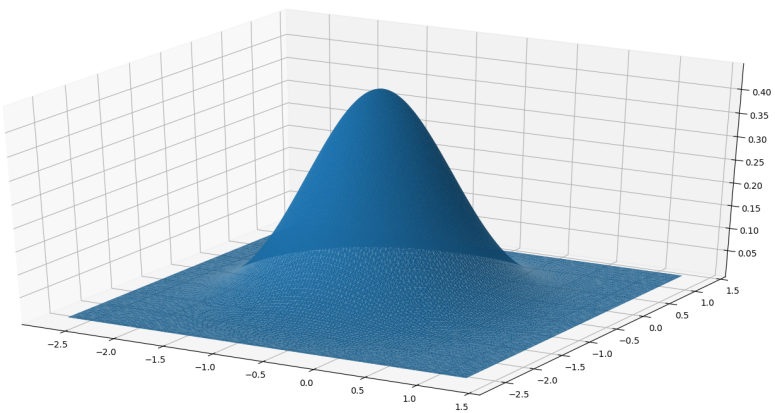
Square-3 $\notin C^2$



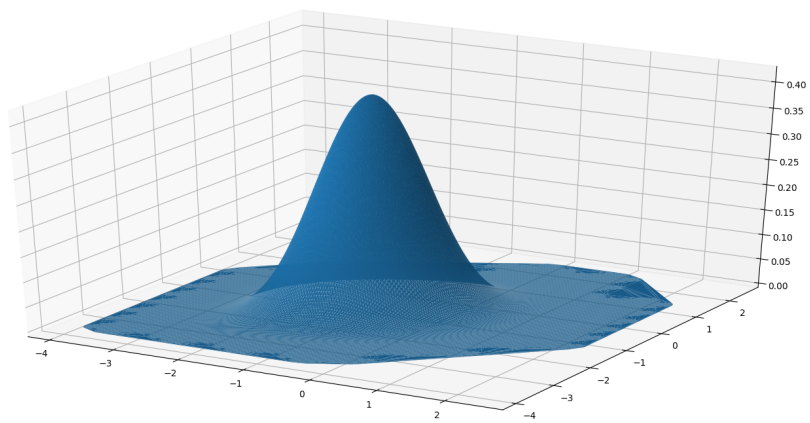
Bear-3 $\in C^2$



Square-4 $\notin C^3$

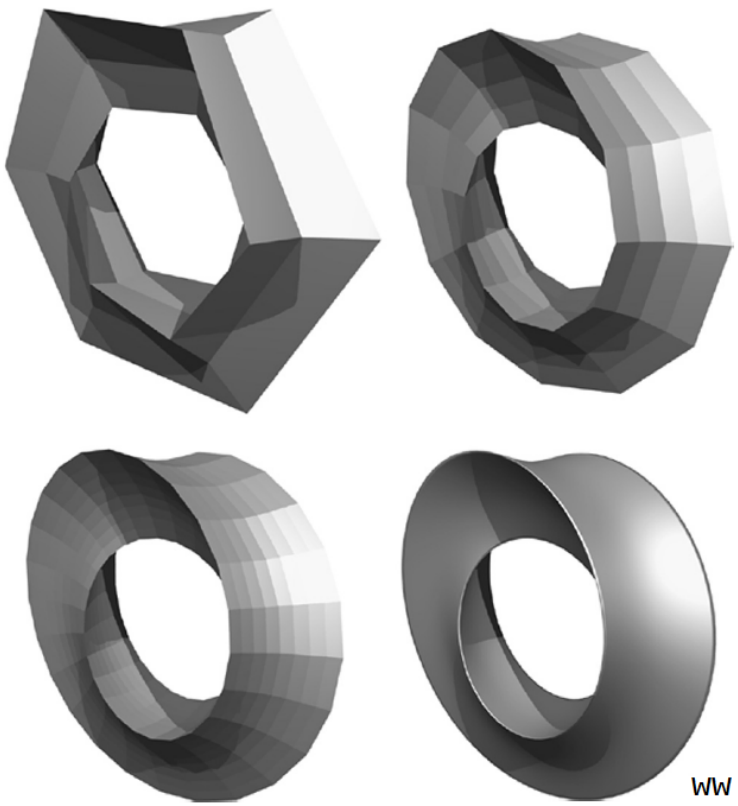


Bear-4 $\in C^3$

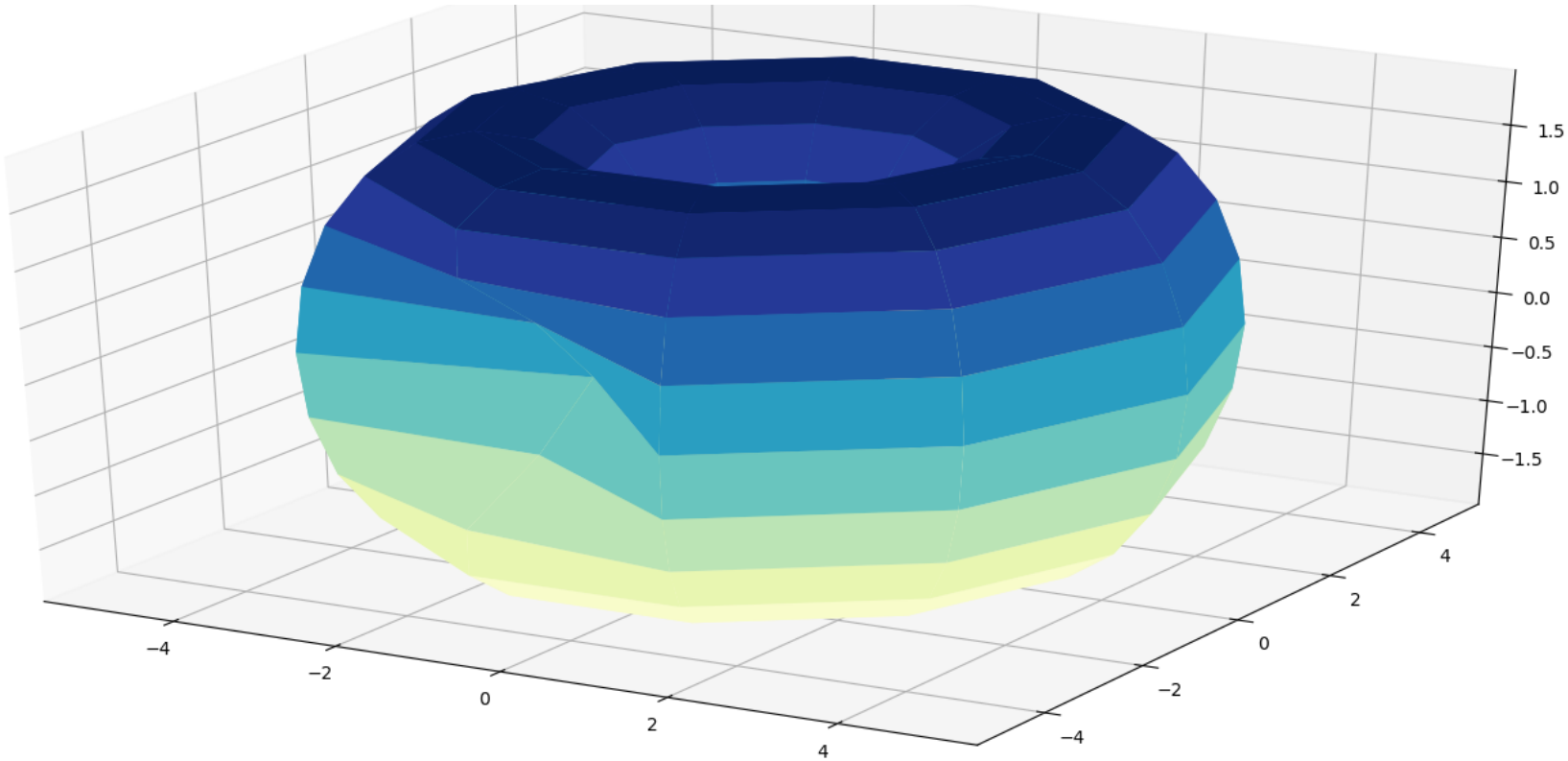


Subdivision schemes

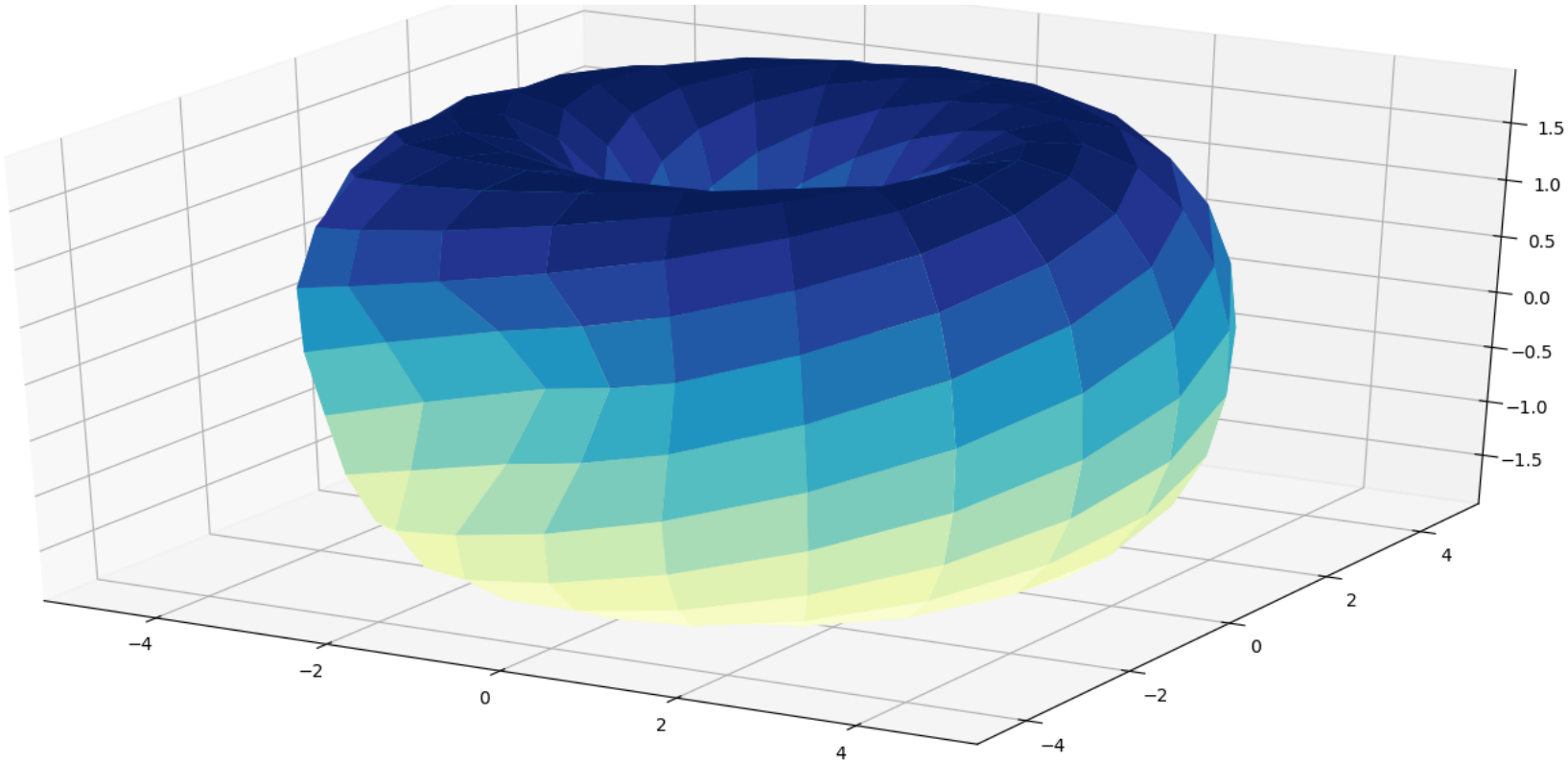
$$[Su](k) = \sum_{j \in \mathbb{Z}^d} c_{k-Mj} \cdot u(j), \quad u \in \ell_\infty.$$



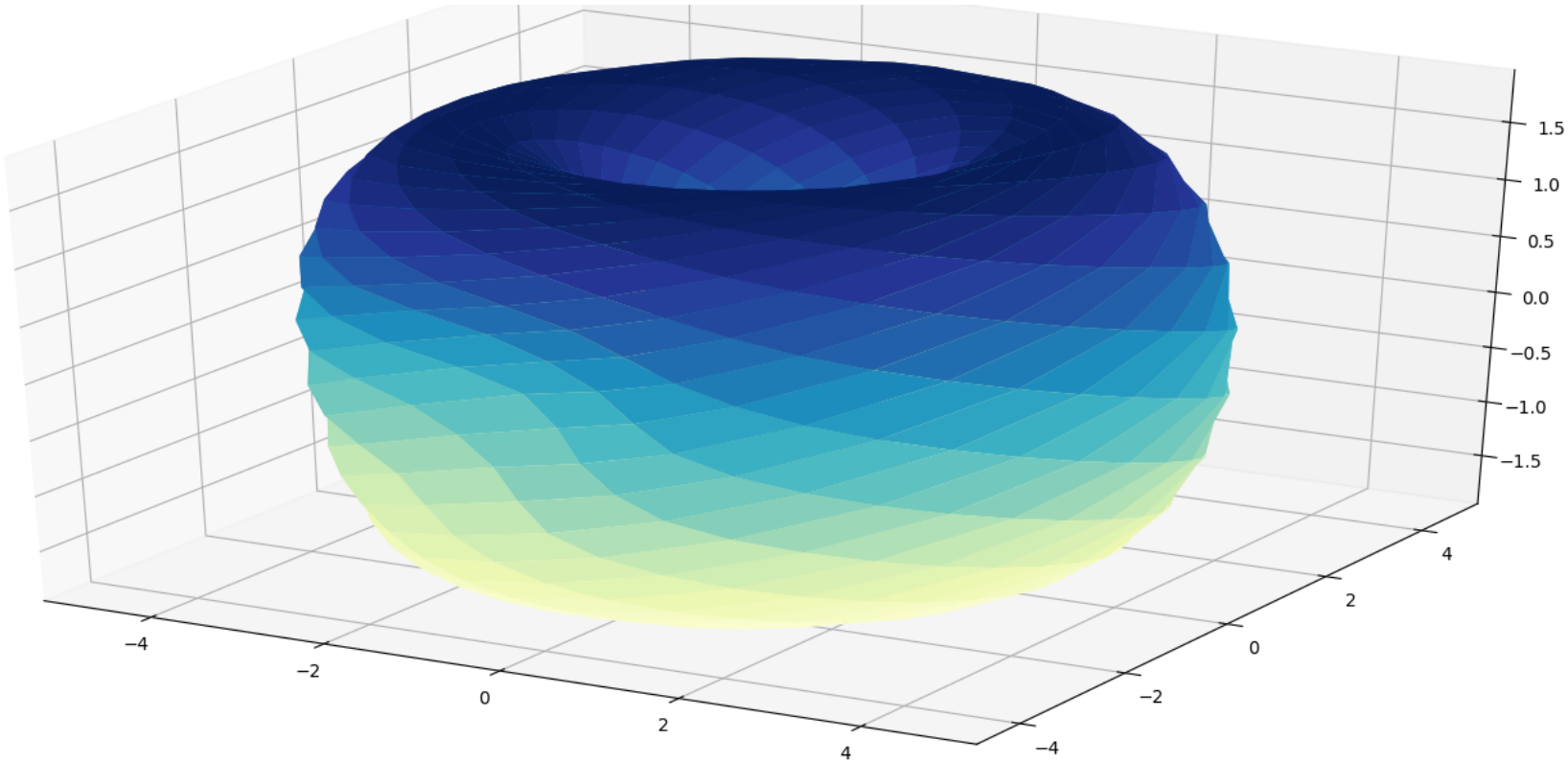
Torus example, iteration 0



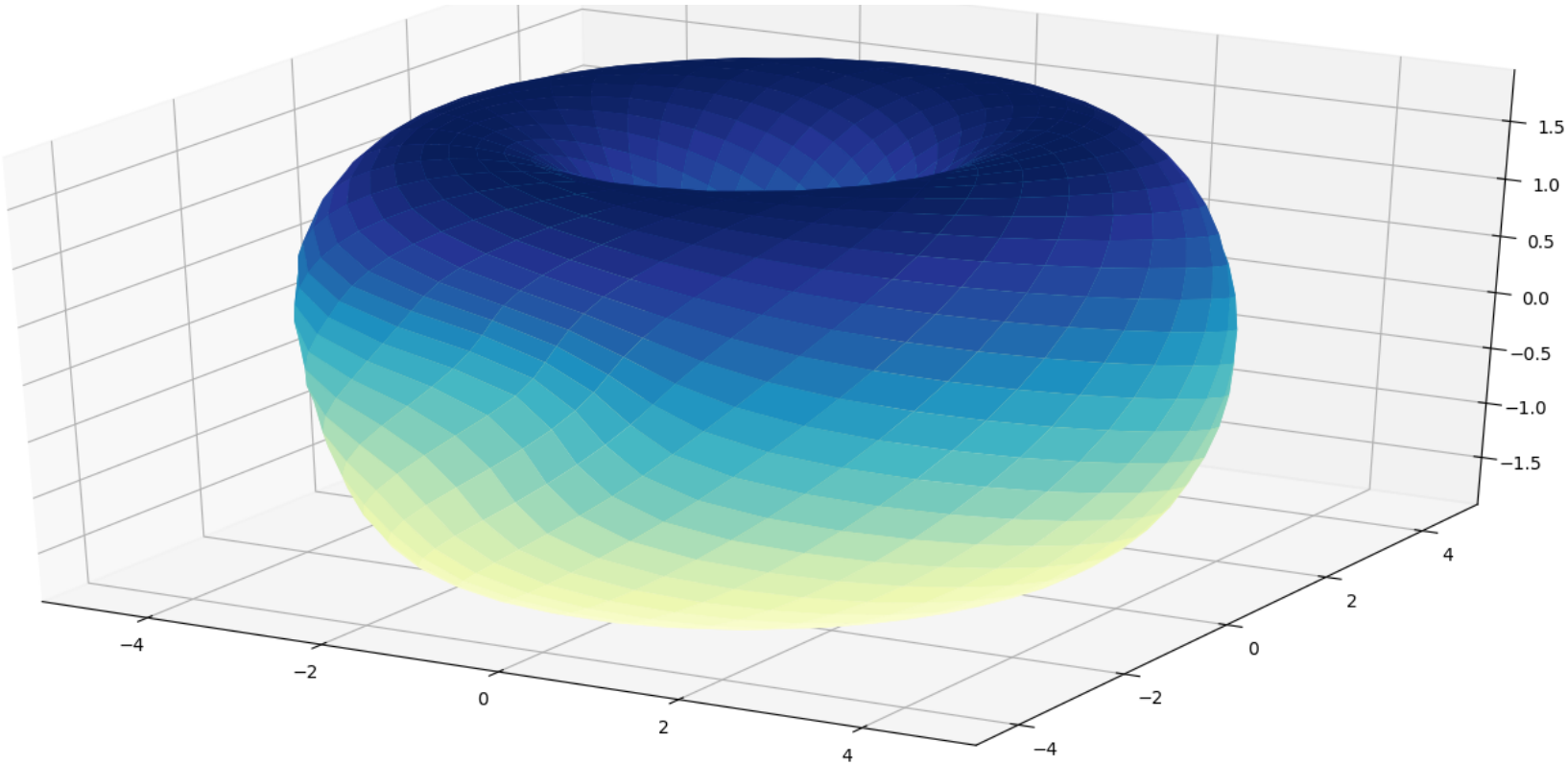
Torus example, iteration 1



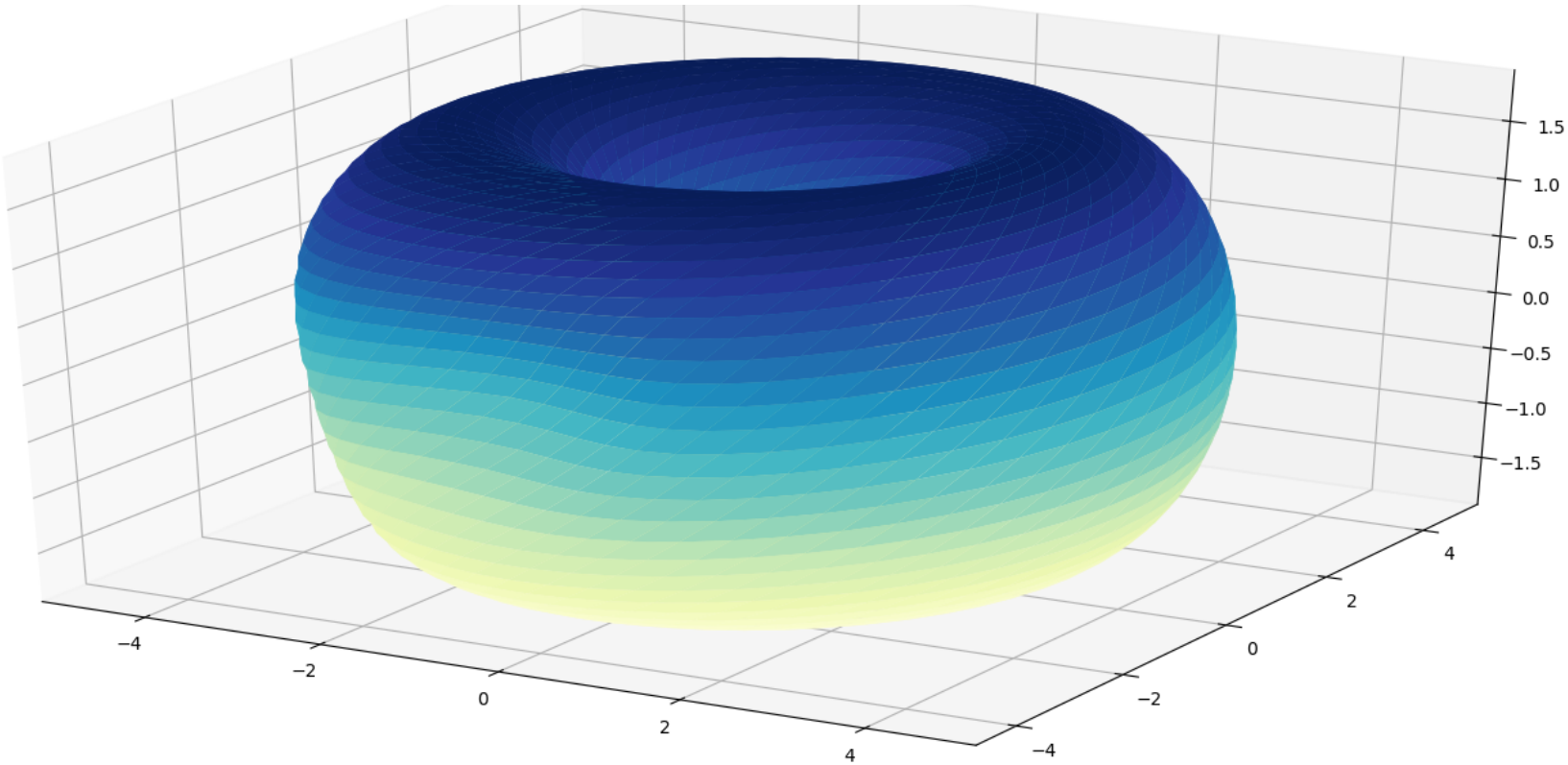
Torus example, iteration 2



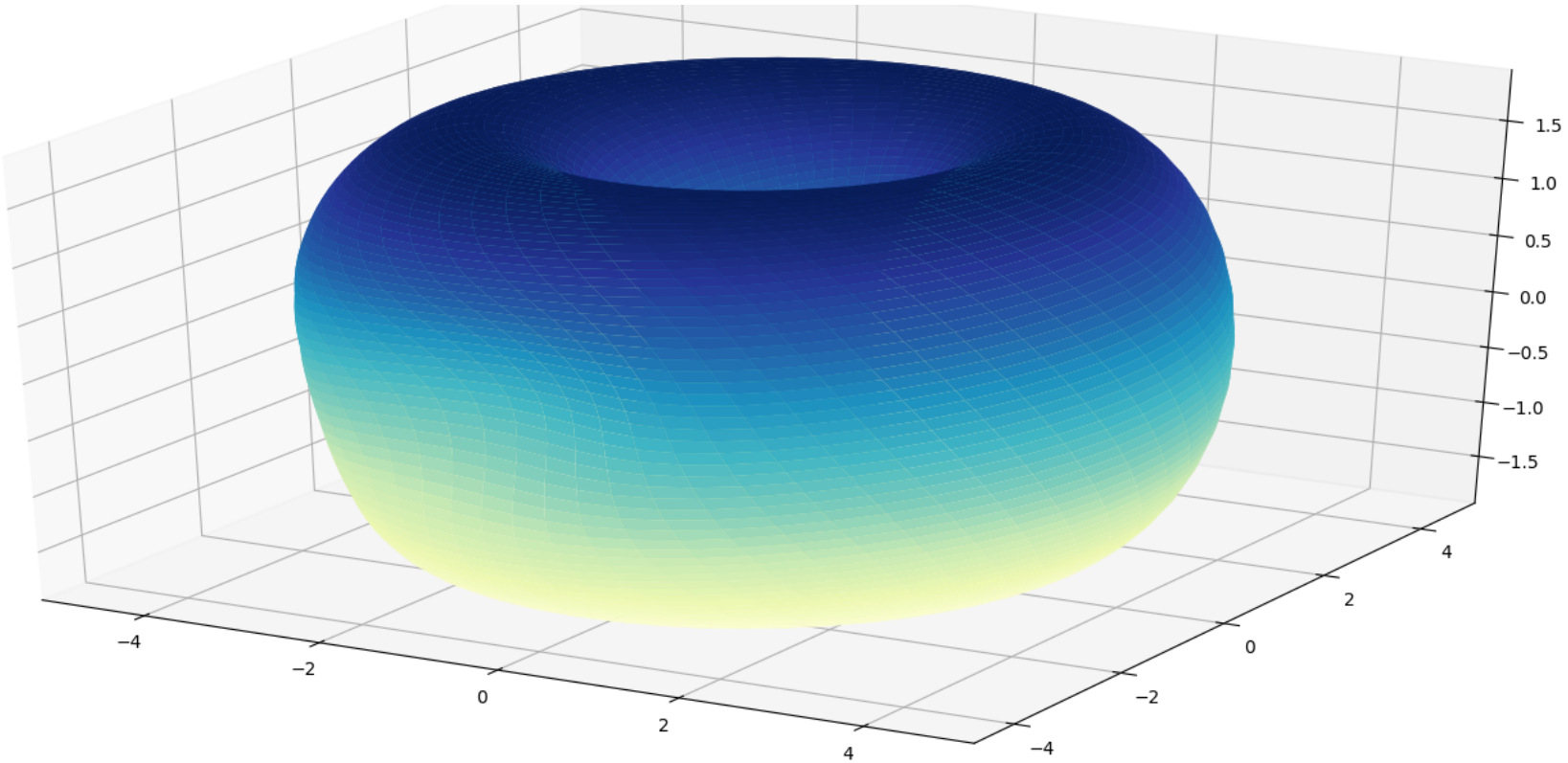
Torus example, iteration 3



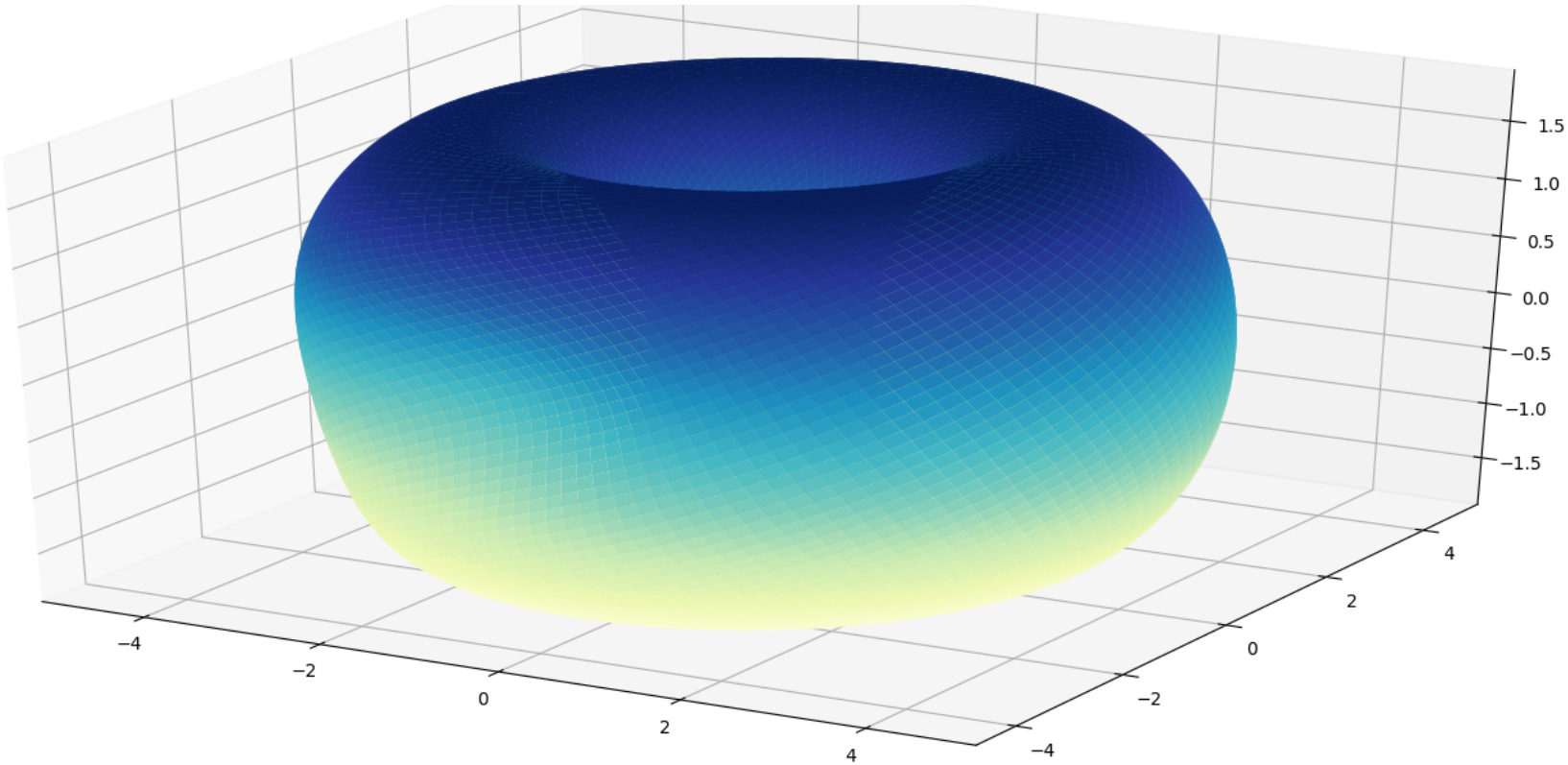
Torus example, iteration 4



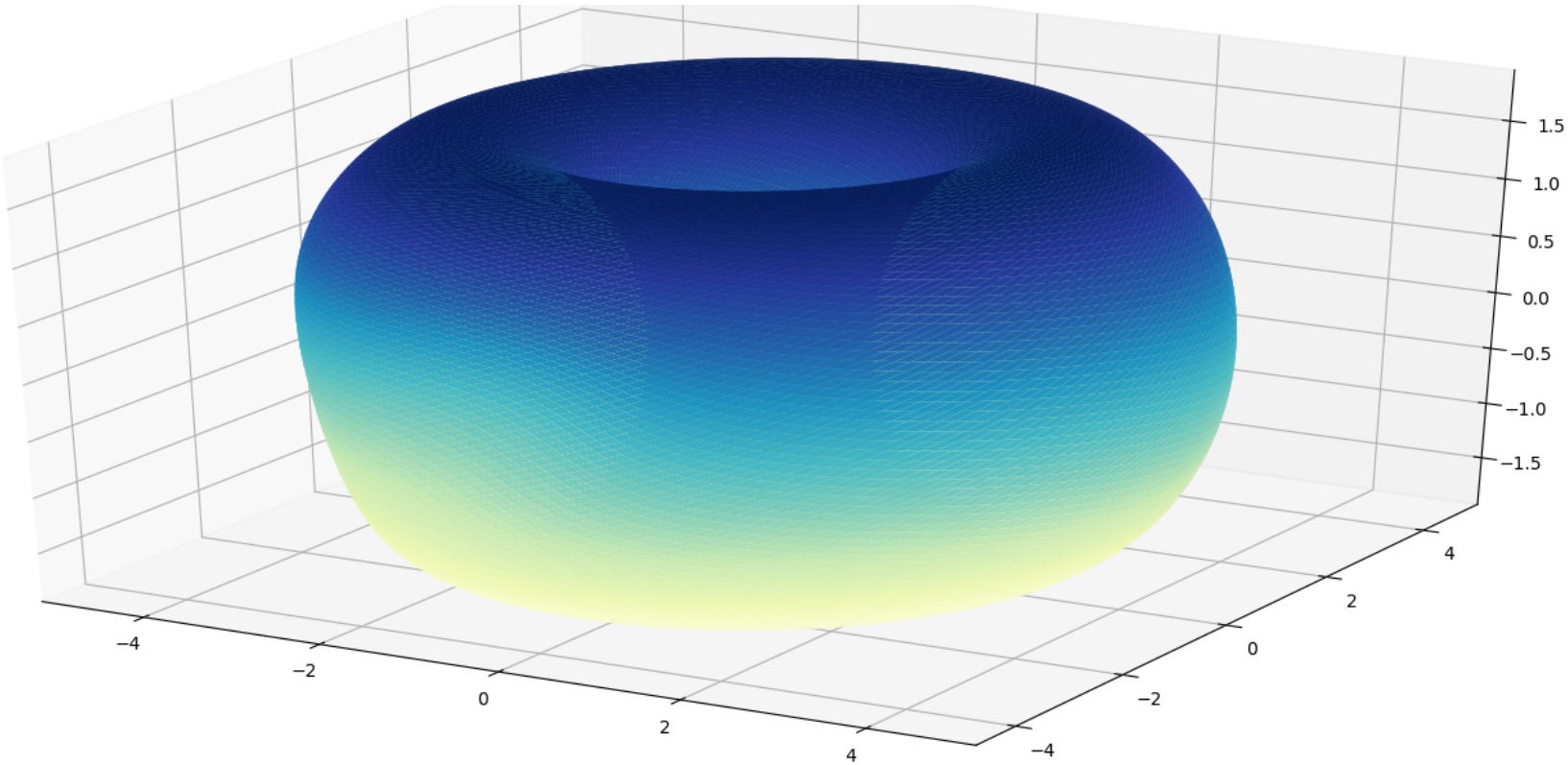
Torus example, iteration 5



Torus example, iteration 6



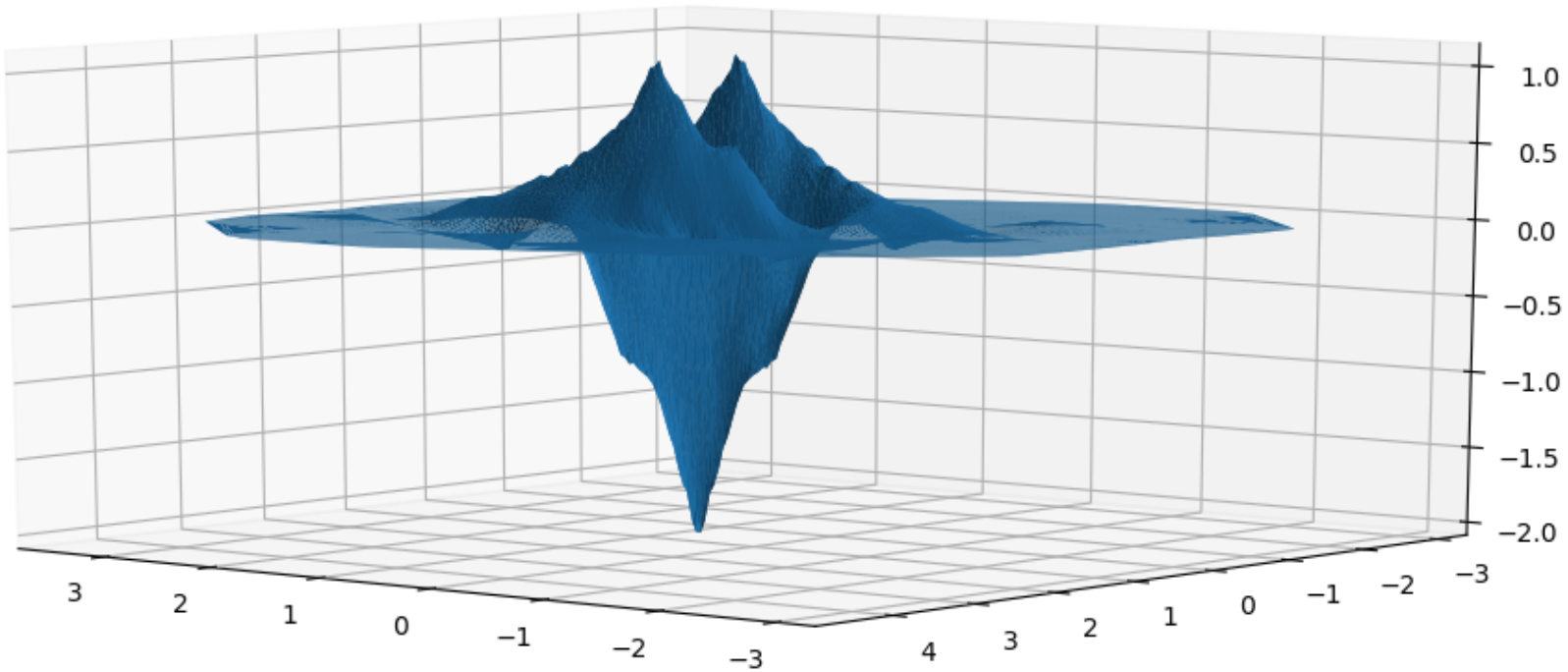
Torus example, iteration 7



More

- the orthogonalization of B-splines
- the construction of wavelet systems
- the estimation of the decay rate of the coefficients of wavelet function

Bear-2 wavelet



Bear-4 wavelet